Instructor: Dr. Siran Li
Office: 444 Herman Brown
Email: Siran.Li@rice.edu

Time: MWF 11:00-11:50AM
Classroom: Herman Brown Hall 423
Office Hours: Tu/W 5-6pm, or by appointment

Class Webpage: https://siranlivictor.wixsite.com/maths

Texts: [dC] M.P. doCarmo, *Riemannian Geometry*, Birkhauser, 1992.
[SHL] G. Sylvestre, D. Hulin, J. Lafontaine, *Riemannian geometry*, Berlin: Springer-Verlag, 1990.
[L] J. M. Lee, *Introduction to Smooth Manifolds*, GTM 218 (More elementary materials).

Course Description: Differential geometry is the study of geometric figures using the methods of calculus. The principal object is a *manifold*, which locally looks like the usual Euclidean space, but globally may be curved *intrinsically*. That is, a manifold can be curved *per se*, rather than being curved in some ambient space. A central notion is that of a Riemannian metric, which provides the concepts of length, angle, volume and various curvatures. In this course we shall begin with basic constructions and operations on differentiable manifolds, and then discuss the metrics, geodesics (generalisation of straight line segments) and curvatures. Equipped with a metric, a differentiable manifold becomes a Riemannian manifold. Next, several comparison theorems will be introduced, which tell us how to compare the length, volume or "shape" of Riemannian manifolds by comparing their curvatures. Moreover, by doing calculus on manifolds, we may also gain some insights for their topology. Finally, time permitted, we shall discuss some physical applications, including the Einstein and Yang-Mills equations.

Course outline: Differentiable Manifolds. Immersions, embeddings. Vector fields, tensor fields. Pushforward and pullback. Integrability. Differential forms. Riemannian metrics. Connections. Geodesics. Exponential map. Lie group and Lie algebra. Riemannian, sectional, Ricci and scalar curvatures. Jacobi fields. Geodesic completeness, Hopf-Rinow theorem. Space forms. Calculus of variations. Hadamard, Bonnet-Myers and Rauch theorems. Morse index. De Rham cohomology. Hodge theorem. Vector and fibre bundles. Some applications in physics.

Prerequisites: Calculus, linear algebra and ODE. Some knowledge about curves and surfaces will be helpful. We may also need some rudiments from group theory and topology.

Assignments: Weekly problem sheets (about 5 questions per week).

Exams: One midterm, one final.

Grading: Problem sheets 40%; Midterm 20%; Final 40%.

Disability support: Any student with a documented disability seeking academic adjustments or accommodations is requested to speak with me during the first two weeks of class. All such discussions will remain as confidential as possible. Students with disabilities will need to also contact Disability Support Services in the Allen Center.

Teaching assistants, recitations and tutoring services: There will be a graduate TA for the course. More information about the TA and his/her office hours will be updated soon.

Contacting me: By email at ⟨Siran.Li@rice.edu⟩, or preferably, come to my office hours on Tuesdays and Wednesdays from 5 to 6pm in 444 Herman Brown, or knock on my door at any time.